Simulating Quantum Field Theories on a Quantum Computer

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Can quantum computers simulate all physical processes efficiently?

Universality Conjecture:
Quantum circuits can simulate all physical dynamics in $\mathrm{poly}(E, V, t, 1/\epsilon)$ time.

Status:

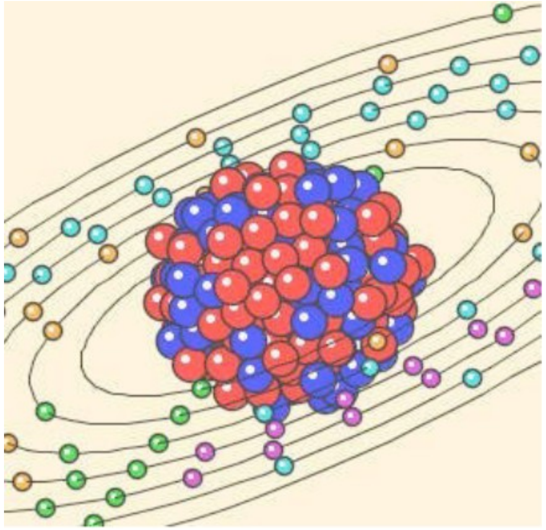
Non-relativistic QM	Yes: Now being optimized
Quantum Field Theories	Probably: In progress
Quantum Gravity/Strings	Nobody knows

Quantum Field Theory

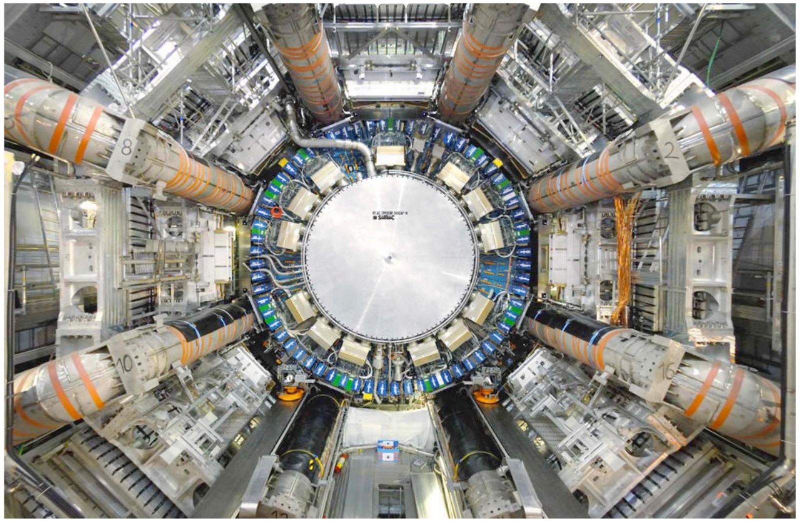
- Much is known about using quantum computers to simulate quantum systems.

- Why might quantum field theory be different?
  - Field has infinitely many degrees of freedom
  - Relativistic
  - Particle number not conserved
  - Formalism looks different.

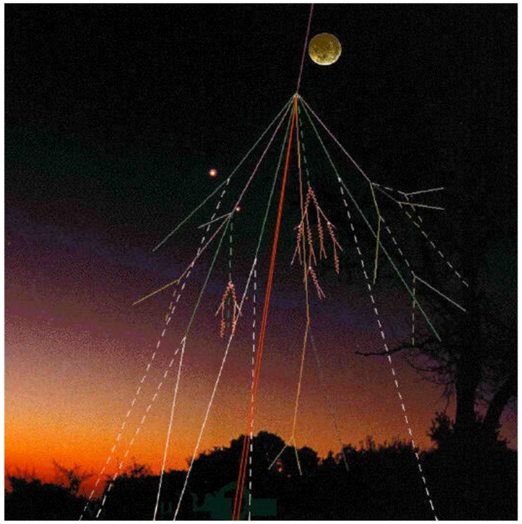
When do we need QFT?



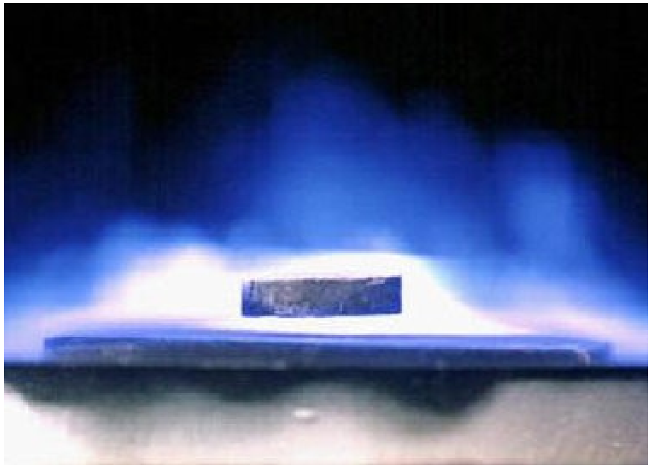
Nuclear Physics



Accelerator Experiments



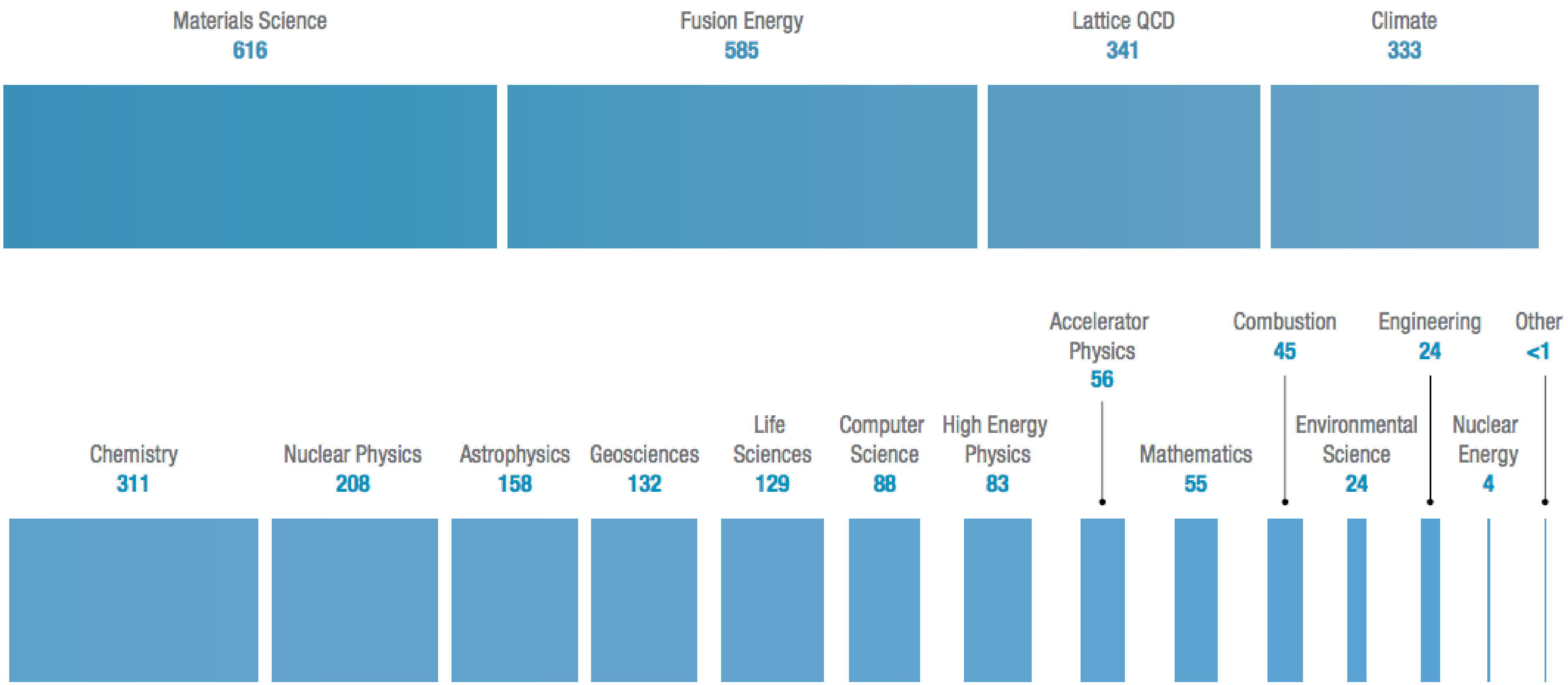
Cosmic Rays



Coarse-grained many-body systems

2015 NERSC Usage By Discipline

(MPP HOURS IN MILLIONS)



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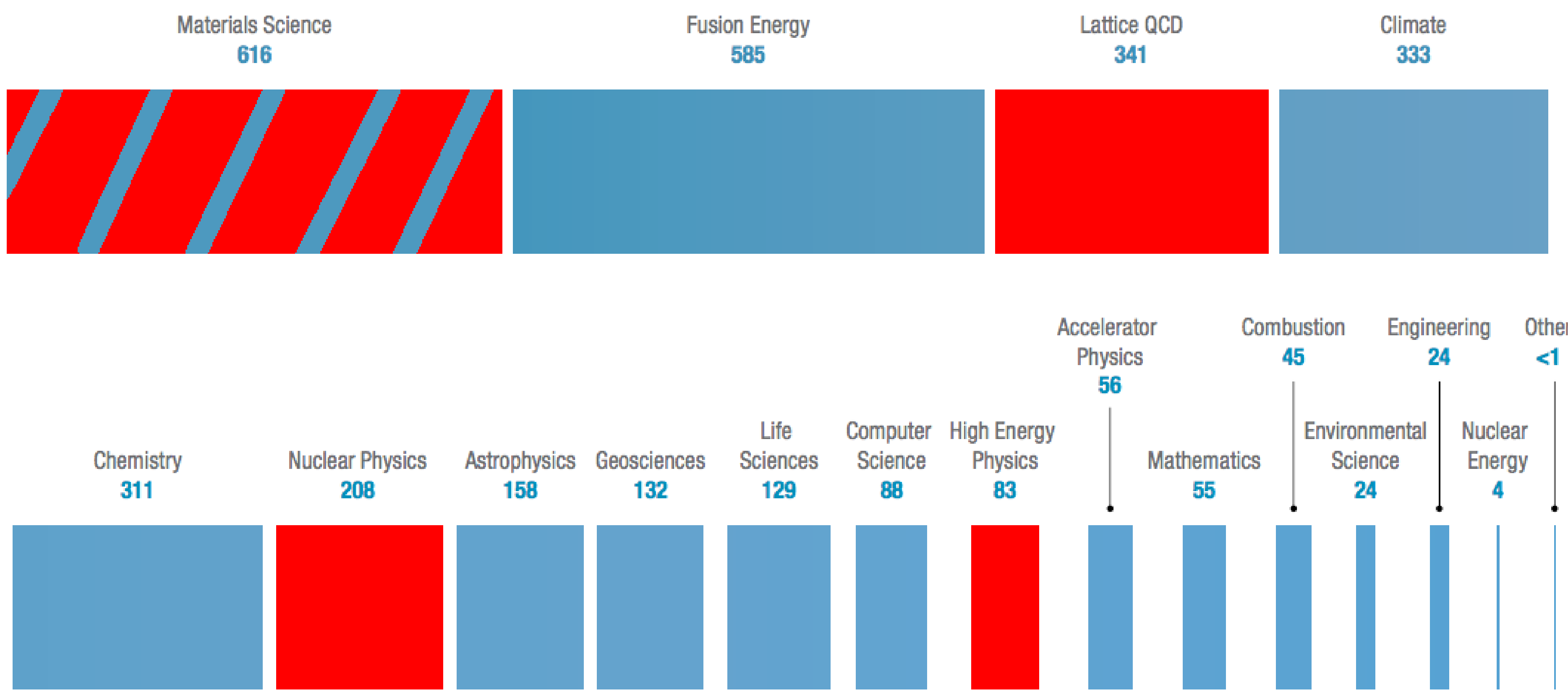
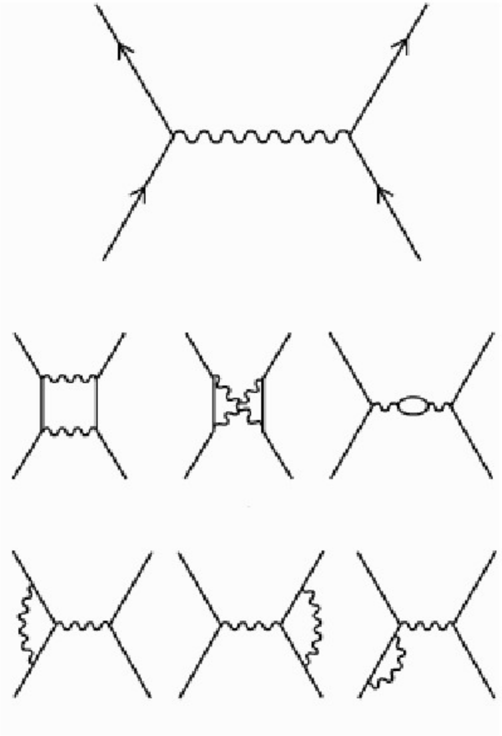


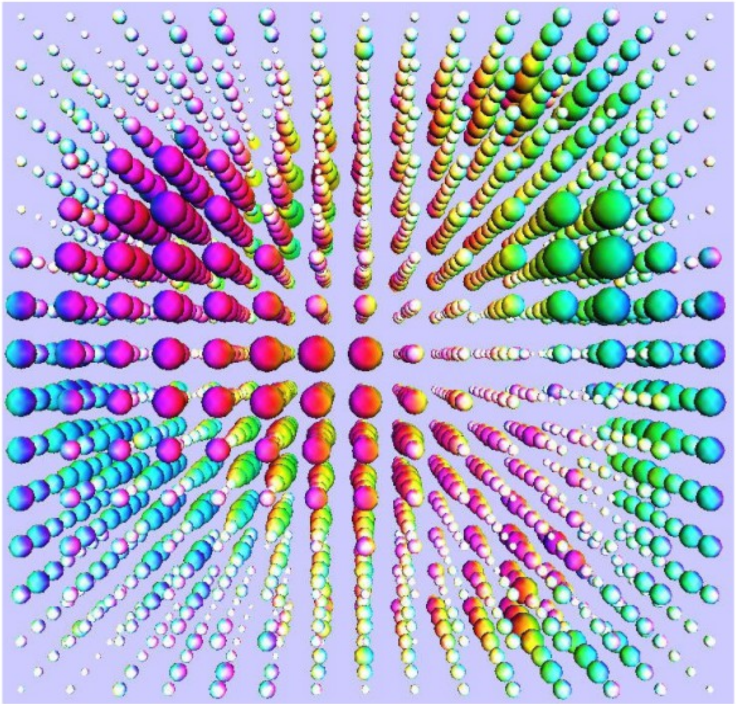
Classical Algorithms

Feynman diagrams



Break down at strong coupling or high precision

Lattice methods



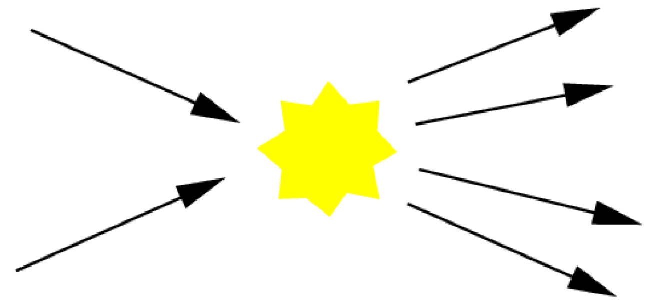
Cannot calculate scattering amplitudes

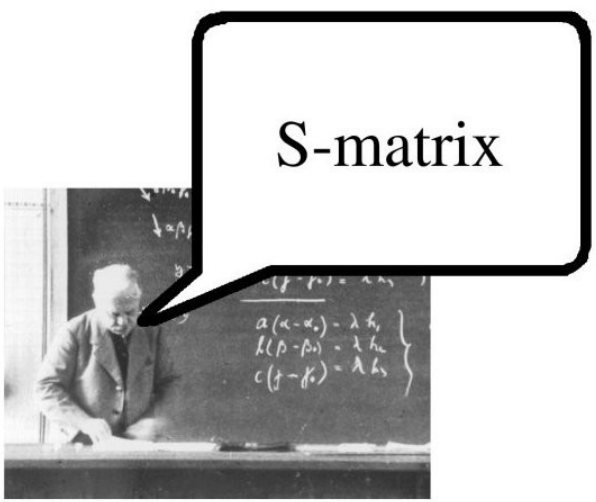
There's room for exponential speedup by quantum computing.

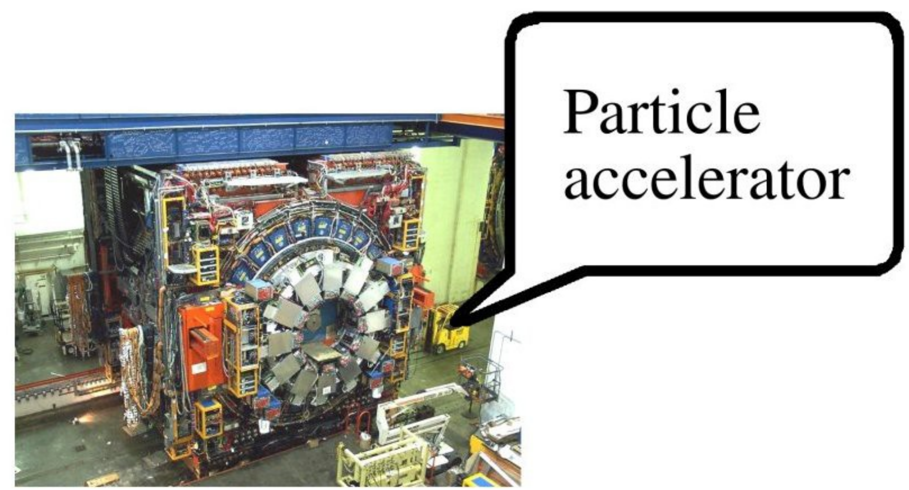
A QFT Computational Problem

Input: a list of momenta of incoming particles.

Output: a list of momenta of outgoing particles.







Results So Far

- Efficient quantum simulation algorithms:

	Bosonic	Fermionic
Massive	Jordan, Lee, Preskill *Science*, 336:1130 (2012)	Jordan, Lee, Preskill *ArXiv:1404.7115* (2014)
Massless	?	?

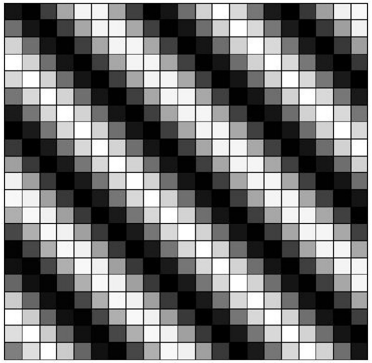
- BQP-hardness: classical computers cannot perform certain QFT simulations efficiently

  [S. Jordan, H. Krovi, K. Lee, K. Preskill, 2017]

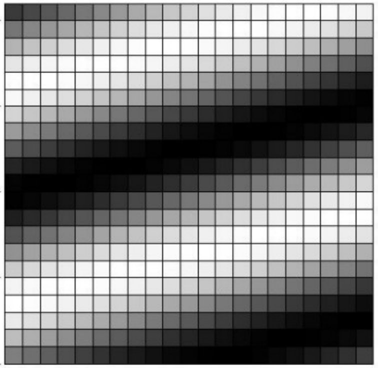
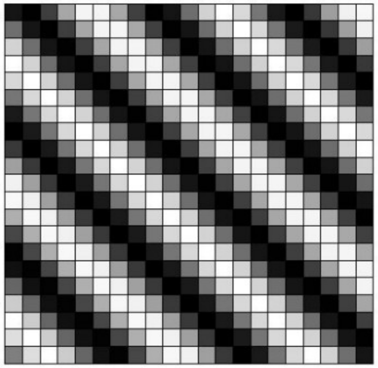
- Better Speed and Broken Symmetries
  [A. Moosavian and S. Jordan, 2017]

Representing Quantum Fields

A field is a list of values, one for each location in space.



A quantum field is a superposition over classical fields.

$\frac{1}{\sqrt{2}}\Big|$  $\Big\rangle - \frac{i}{\sqrt{2}}\Big|$  $\Big\rangle$

A superposition over bit strings is a state of a quantum computer.

ϕ^4-theory

Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi \partial_\mu \phi - m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

For quantum simulation we prefer Hamiltonian formulation (equivalent)

$$H = \int d^d x \left[\pi^2 + (\nabla\phi)^2 + m^2\phi^2 + \lambda\phi^4\right]$$

$$[\phi(x), \pi(y)] = i\delta^{(d)}(x-y)$$

Our Algorithms

1) Choose a lattice discretization.

    Bound discretization error (by renormalization group).

2) Prepare physically realistic initial state.

    Is the most time-consuming step.

    This depends strongly on which QFT simulated.

3) Implement time-evolution by a quantum circuit.

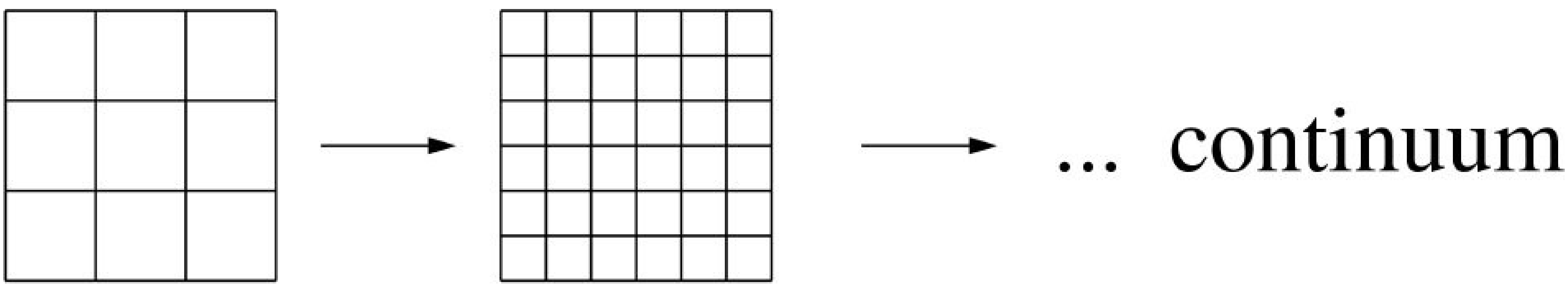
    Use Trotter formulae.

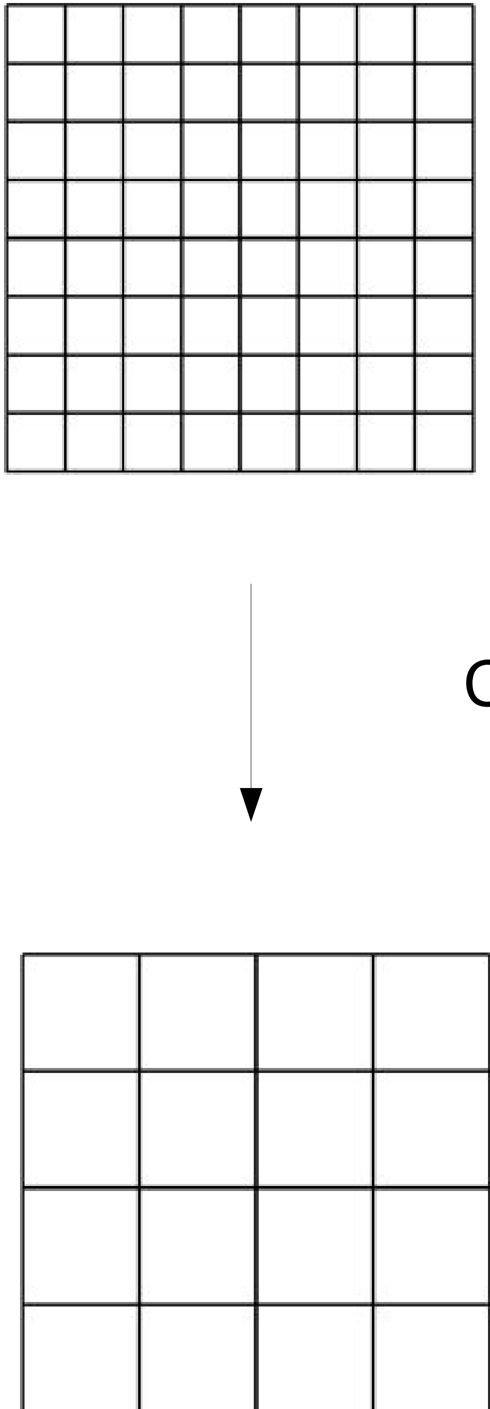
4) Perform measurements on final state.

    Complicated by vacuum entanglement.

Lattice Cutoff

Continuum QFT = limit of a sequence of theories on successively finer lattices.





Mass: m

Interaction strength: λ

Coarse grain

Mass: m'

Interaction strength: λ'

Lattice Cutoff

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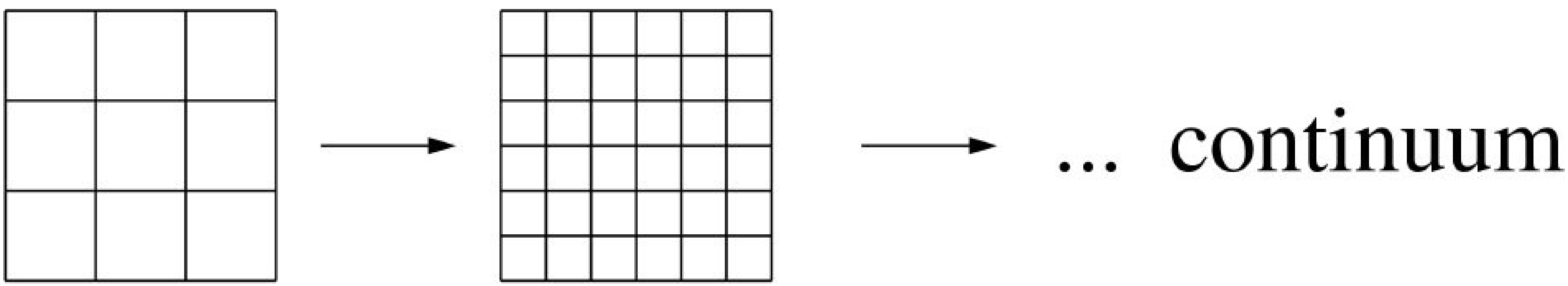
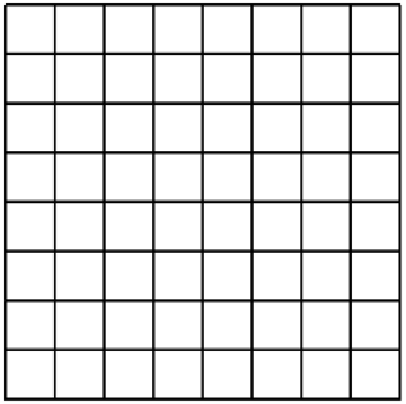
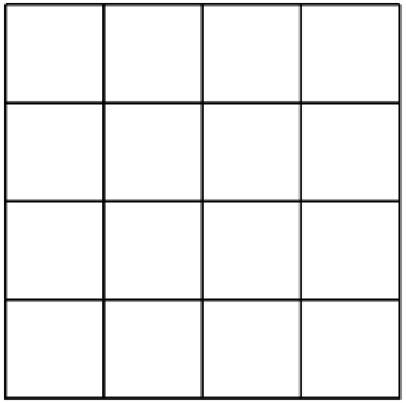


m and λ are functions of lattice spacing!



$$H = \frac{1}{2}\sum_{x\in\Omega} a^d \left[\pi^2 + (\nabla\phi)^2 + m^2\phi^2 + \lambda\phi^4\right]$$

Coarse grain



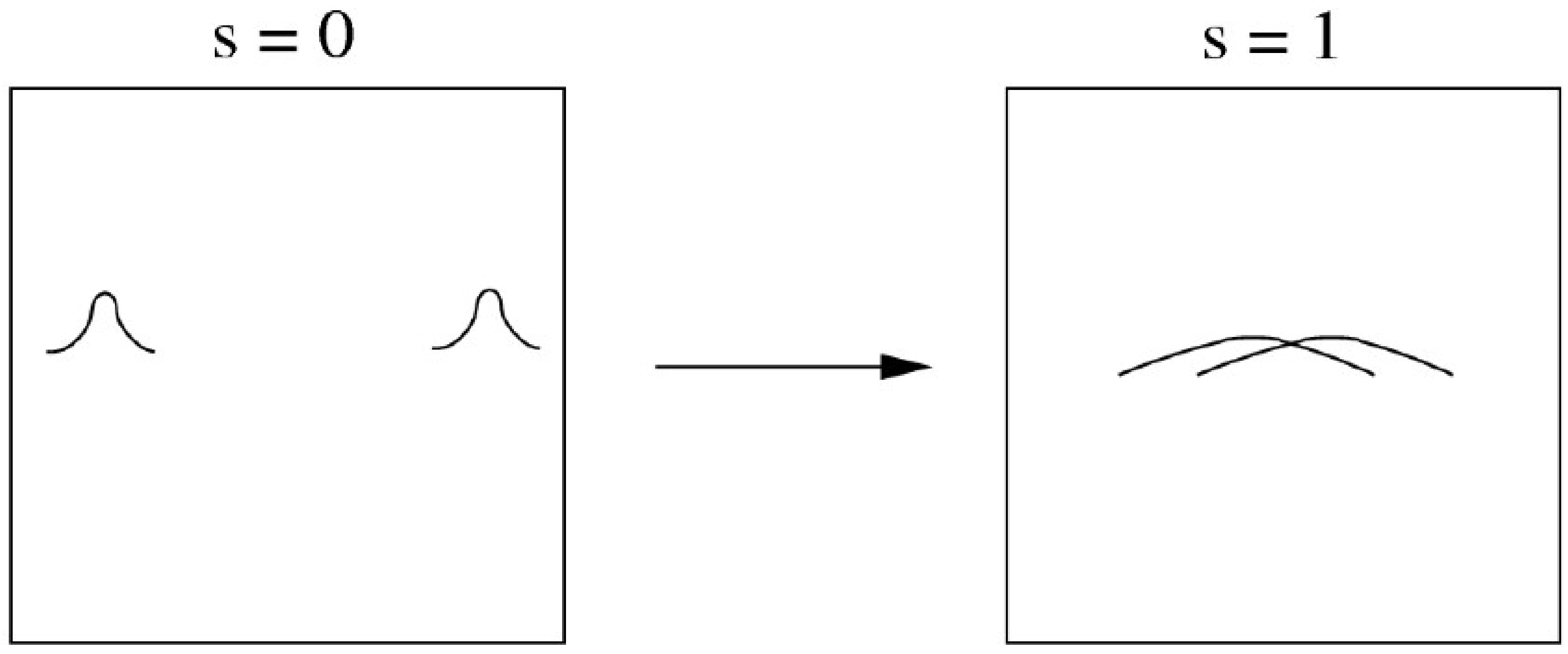
$$H_{\rm eff} = \frac{1}{2}\sum_{x\in\Omega'} (2a)^d \left[\pi^2 + (\nabla'\phi)^2 + m_{\rm eff}^2\phi^2 + \lambda_{\rm eff}\phi^4 + \boxed{g\phi^6 + \ldots}\right]$$

Simulation converges as a^2

Adiabatic State Preparation

$$H(s) = H_{\text{free}} + sH_{\text{interaction}}$$

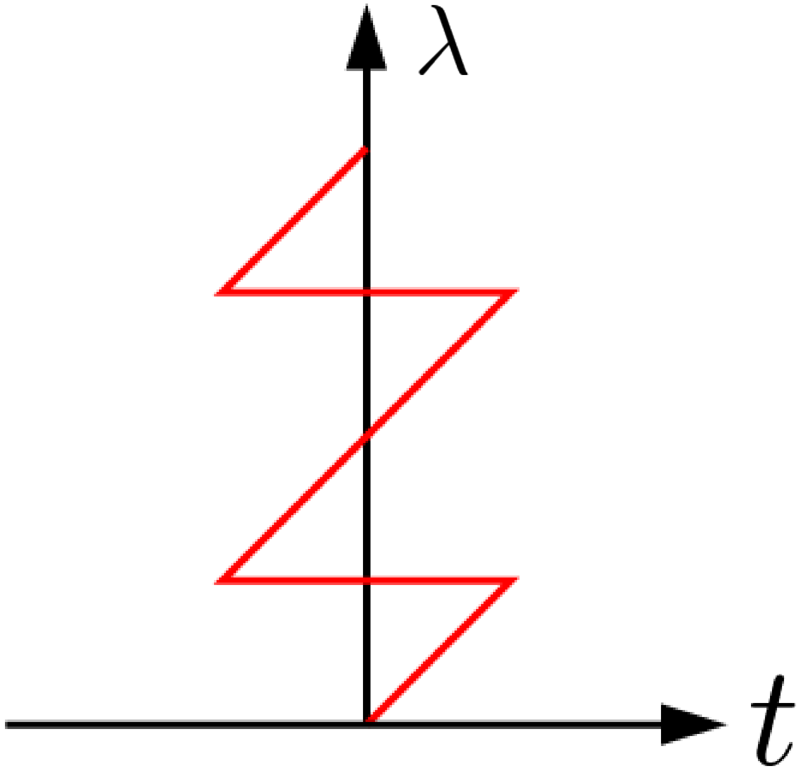
Prepare wavepackets in free theory, then adiabatically turn on interaction. Problem:



Adiabatic State Preparation

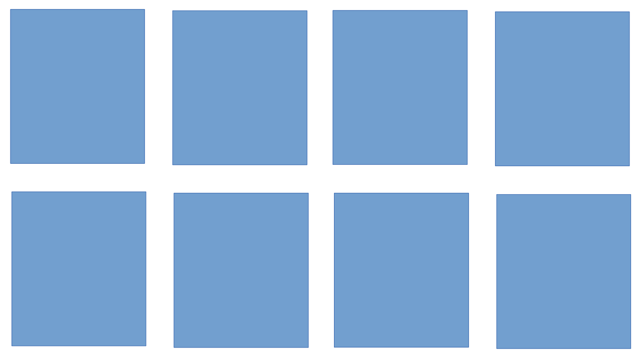
Solution: intersperse backward time evolutions with time-independent Hamiltonians.

This winds back dynamical phase on each eigenstate without undoing adiabatic change of basis.



Simulating Detectors

- Measure energy in localized regions:



- Need smooth envelope function to avoid excessive vacuum noise!

$$H_f = \sum_{\mathbf{x}} f(\mathbf{x})\mathcal{H}(\mathbf{x})$$

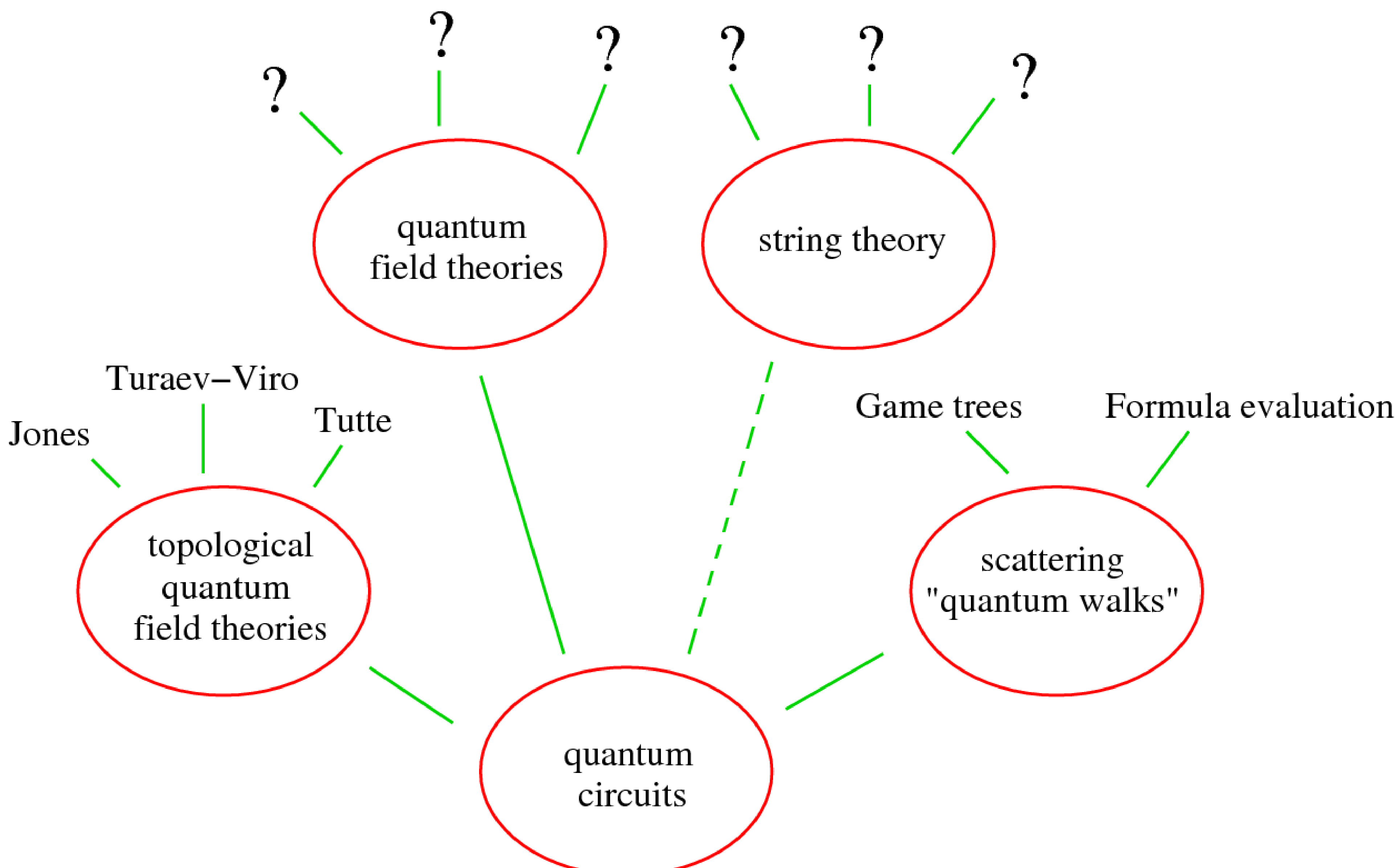
Runtimes

Weak Coupling:

$d=1$	$(1/\epsilon)^{1.5}$
$d=2$	$(1/\epsilon)^{2.376}$
$d=3$	$(1/\epsilon)^{5.5}$

Strong Coupling:

	$\lambda_c-\lambda_0$	p	n_{out}
$d=1$	$\left(\frac{1}{\lambda_c-\lambda_0}\right)^{9}$	p^4	n_{out}^5
$d=2$	$\left(\frac{1}{\lambda_c-\lambda_0}\right)^{6.3}$	p^6	$n_{\text{out}}^{7.128}$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?
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quantum
field theories
string theory
Turaev−Viro
Jones
Tutte
Game trees
Formula evaluation
topological
quantum
field theories
scattering
"quantum walks"
quantum
circuits

Fermions:

- Fermion doubling problem
- Free vacuum different from Bosonic case

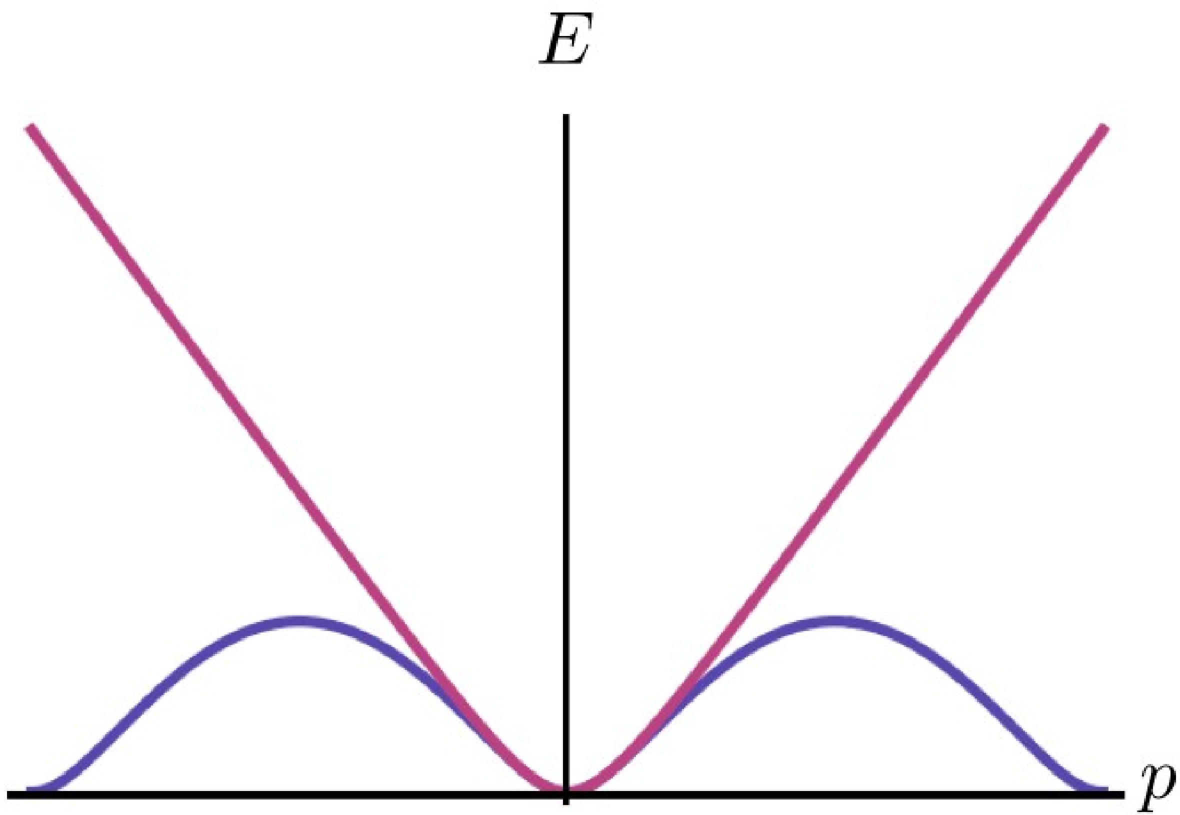
Gross-Neveu:

$$H = \int dx \left[\sum_{j=1}^{N} \bar{\psi}_j \left(m_0 - i\gamma^1 \frac{d}{dx} \right) \psi_j + \frac{g^2}{2} \left(\sum_{j=1}^{N} \bar{\psi}_j \psi_j \right)^2 \right]$$

Fermion Doubling Problem

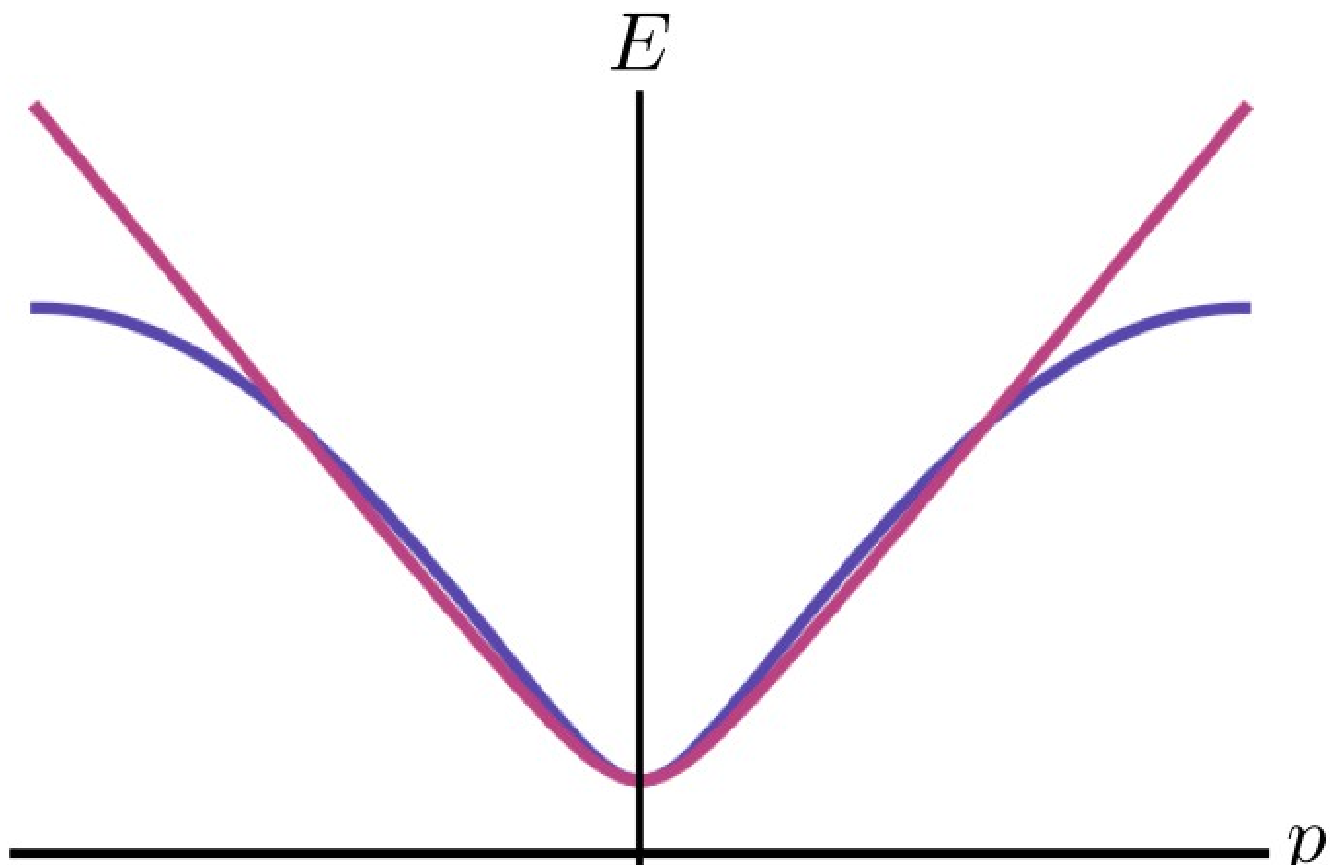
$$\frac{d\psi}{dx} \to \frac{\psi(x+a)-\psi(x-a)}{2a}$$

$$\sqrt{p^2+m^2} \to \sqrt{\sin^2 p + m^2}$$



Wilson Term

$$H \to H - \frac{r}{2a} \sum_x \bar{\psi} \left(\psi(x+a) - 2\psi(x) + \psi(x-a) \right)$$



Improved State Prep: Bosons

- In some cases (e.g. weakly coupled d=2), preparing the free vacuum is the rate limiting step.

- We can do this much faster using Bogoliubov transformation that looks like a Fast Fourier Transform.
  [Somma, Jordan, unpublished]

- Essentially same idea as 2nd quantized FFT from:

  [Babbush, Wiebe, McClean, McLain, Neven, Chan, 2017]

Improved State Prep: Fermions

- Two problems with adiabatic state preparation:
  - Cannot reach symmetry-broken phase
  - Runtime bound not practical: $O(\epsilon^{-8})$
- A solution for both:
  - First, prepare the vacuum from MPS
  - Then, resonantly excite single-particle wavepackets
  - Tighter analysis: CFT entropy and Floquet theory: $O(\epsilon^{-3.23})$

[A. Moosavian, S. Jordan, 2017]

Tensor Network Ansatzes

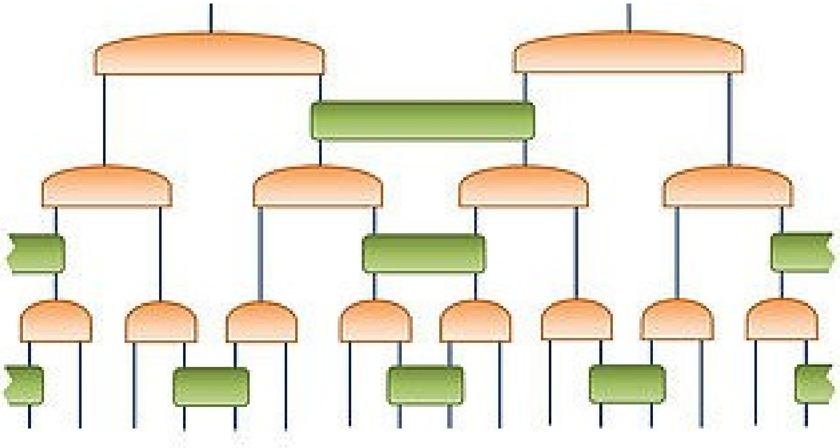
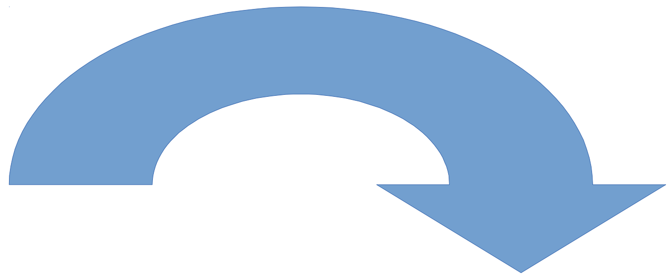
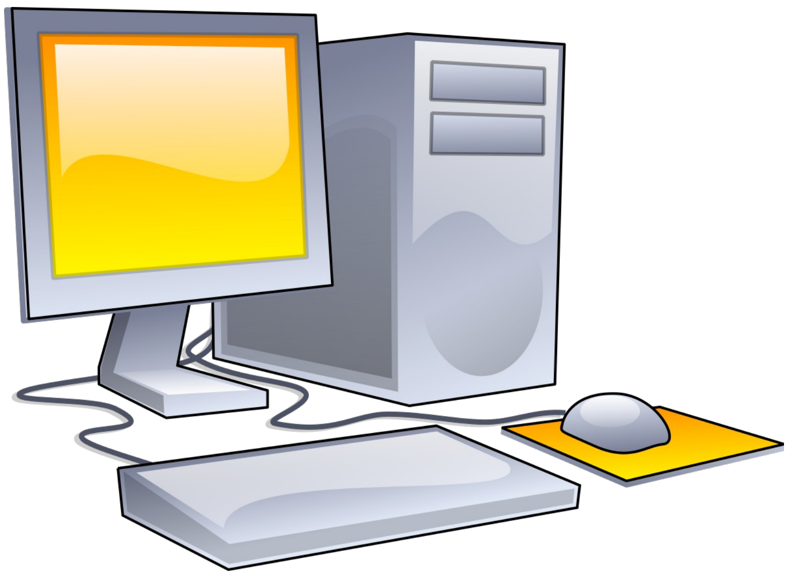
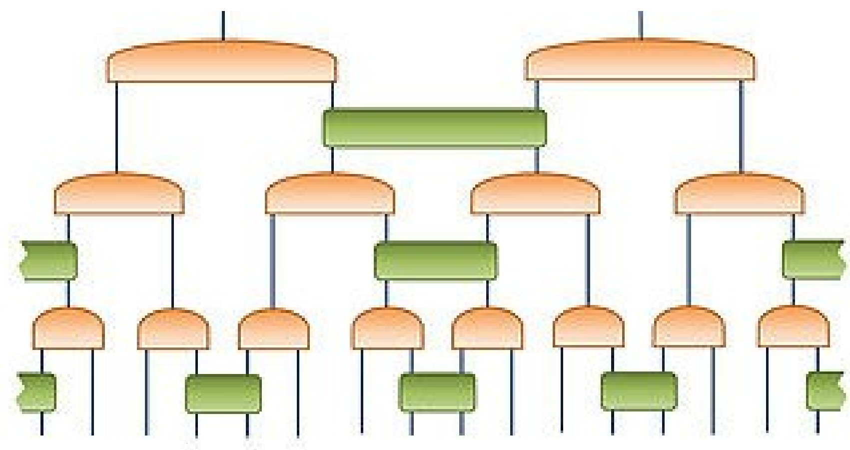


image credit: G. Evenbly

Tensor Network Ansatzes







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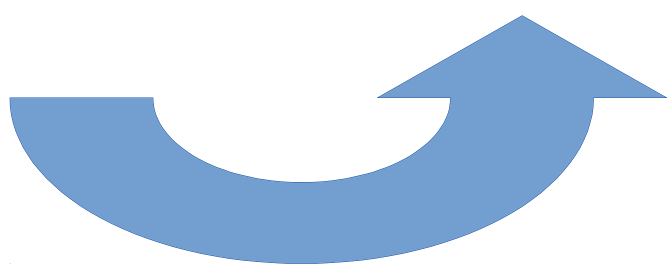

[Swingle, Kim, 2017]

Near-Term Prospects?

quantum supremacy → science applications → commercial applications

- Simulating conformal field theories using MERA-based variational eigensolvers
- Simulating commuting Hamiltonians
- Simulating high-connectivity systems, e.g. spin glasses or SYK model

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Thanks!